Year: 12 Subject: Further Maths A level	**Curriculum Intent:** Some pure topics from A Level mathematics are studied in greater depth, while some new topics are introduced. Algebraic work with series is extended. The powerful technique of proof by induction is used in various contexts. Complex numbers are introduced and lead to solutions of problems in algebra, geometry and trigonometry. Matrices are used to solve systems of equations and to explore transformations. Vector methods are applied to problems involving lines and planes. In statisrtics situations are modelled by discrete random variables; the suitability of models is tested using Chi Squared tests. Bivariate data are investigated, with tests for correlation and association and modelling using regression. Algorithms play a central part in the modern world. Modelling with algorithms explores them in their own right. Algorithms can be run by hand, but when algorithms are used to model real world problems then technology allows their application to authentic problems. A range of optimisation and network problems are introduced. Many of these can be formulated as linear programming problems, allowing them to be solved using technology.					
	Term 1		**Term 2**			
Topic Titles (in order of delivery)	1. Complex Numbers 2. Matrices	1. Roots of Polynomials 2. Sequences and Series 3. Vectors 4. Algorithms	1. Statistical Problem Solving 2. Discrete Random Variables 3. Discrete Probability Distributions 4. Bivariate, Measuring Correlation	1. Modelling with Graphs and Networks 2. Network Algorithms 3. Further Network Problems 4. Linear Programming 5. Simplex Method	1. Bivariate, Linear Regression 2. Bivariate, Chi Squared Tests 3. Matrices 4. Vectors	1. Reformulating Network Problems and Linear Programming Problems 2. Forces and Motions 3. A Model for Friction 4. Dimensional Analysis 5.
Key knowledge / Retrieval topics	1. **Complex Numbers**: • Understand the language of complex numbers. • Be able to solve any quadratic equation with real coefficients. • Know that the complex roots of polynomial equations with real	**1. Roots of Polynomials:** • Understand and use the relationships between the roots and coefficients of quadratic, cubic and quartic equations. • Be able to form a new equation whose	**1. Statistical Problem Solving** • Be able to explain the importance of sample size in experimental design. • Be able to explain why sampling may be necessary in	1. **Modelling with Graphs and Networks**: • Understand and be able to use graphs and associated language. • Be able to model problems by using graphs.	1. **Bivariate, Linear Regression**: • Be able to calculate the equation of the least squares regression line using raw data or summary statistics. • Be able to use the regression line as a	**1. Reformulating Network Problems and Linear Programming Problems** • Recognise when an LP requires an integer solution.

	coefficients occur in conjugate pairs. Be able to solve cubic or quartic equations with real coefficients. • Be able to add, subtract, multiply and divide complex numbers given in the form $x + yi$, x and y real. • Understand that a complex number is zero if and only if both the real and imaginary parts are zero. • Be able to use radians in the context of complex numbers. • Be able to represent a complex number in modulus-argument form. Be able to convert between the forms $z = x + yi$ and $z = r(\cos\theta + i\sin\theta)$ where r is the modulus and θ is the argument of the complex number. • Be able to multiply and divide complex numbers in modulus-argument form. • Be able to represent and interpret complex numbers and their conjugates on an Argand diagram. • Be able to represent the sum, difference, product and quotient of two complex numbers on an Argand diagram.	roots are related to the roots of a given equation by a linear transformation. **2. Sequences and Series:** • Be able to use standard formulae for Σr , $\Sigma r^2 and\ \Sigma r^3$ and the method of differences to sum series. • Be able to construct and present a proof using mathematical induction for given results for a formula for the nth term of a sequence, the sum of a series or the nth power of a matrix. 3. **Vectors**: • Know how to calculate the scalar product of two vectors, and be able to use the two forms of thescalar product to find the angle between two vectors. • Be able to form and use the vector and cartesian equations of a plane. Convert between vector and cartesian forms for the equation of a plane. • Know that a vector which is perpendicular to a	order to obtain information about a population, and give desirable features of a sample. • Be able to explain the advantage of using a random sample when inferring properties of a population. 2. **Discrete Random Variables**: • Be able to use probability functions, given algebraically or in tables. • Be able to calculate the numerical probabilities for a simple distribution. • Be able to draw and interpret graphs representing probability distributions. • Be able to calculate the expectation (mean), $E(X)$, and understand its meaning. • Be able to calculate the variance, $Var\ (X)$, and understand its meaning. • Be able to use the result $E(a + bX) = a + bE(X)$	• Understand that a network is a graph with weighted arcs. • Be able to model problems by using networks. **2. Network Algorithms** • Be able to solve minimum connector problems using • Kruskal's and Prim's algorithms. • Model shortest path problems and solve using Dijkstra's algorithm. • Know and use the fact that Kruskal's, Prim's and • Dijkstra's algorithms have quadratic complexity. 3. **Further Network Problems**: • Model precedence problems with an activity-on-arc network. • Use critical path analysis and be able to interpret outcomes, including implications for criticality. • Be able to analyse float (total, independent and interfering),	model to estimate values and know when it is appropriate to do so. • Know the meaning of the term residual and be able to calculate and interpret residuals. • Be able to calculate the equation of the two least squares regression lines, y on x and x on y, using raw data or summary statistics. • Be able to use either regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so. • Check how well the model fits the data. • Know the relationship between the two regression lines and when to use one rather than the other. • Be able to use the correct regression line to estimate the expected value of one variable for a	• Be able to formulate a range of network problems as LPs. **2. Forces and Motions** • Understand the language relating to forces. Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative • Be able to resolve a force into components and be able to select suitable directions for resolution. • Be able to find the resultant of several concurrent forces by vector addition. • Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero. • Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium. • Be able to formulate and solve equations for equilibrium by

	• Be able to represent and interpret sets of complex numbers as loci on an Argand diagram. 1. **Matrices**: • Be able to add, subtract and multiply conformable matrices, and to multiply a matrix by a scalar. • Understand and use the zero and identity matrices, understand what is meant by equal matrices. • Know that matrix multiplication is associative but not commutative. • Be able to find the matrix associated with a linear transformation and vice versa. • Understand successive transformations in two dimensions and the connection with matrix multiplication. • Understand the language of vectors in two dimensions and three dimensions. • Be able to calculate the determinant of a 2 × 2 matrix and a 3 × 3 matrix. • Know the meaning of the terms singular and non- singular as applied to matrices. • Know that the magnitude of the determinant of a 2 × 2	plane is perpendicular to any vector in the plane. • Know the different ways in which three distinct planes can be arranged in 3-D space. • Be able to solve three linear simultaneous equations in three variables by use of the inverse of the corresponding matrix. • Interpret the solution or failure of solution geometrically in terms of the arrangement of three planes. • Be able to find the intersection of three planes when they meet in a point. • Know that the angle between two planes can be found by considering the angle between their normals. **4. Algorithms** • Understand that an algorithm is a finite sequence of • operations for carrying out a procedure or solving a problem. • Understand that an algorithm can be the basis for a • computer program.	and understand its meaning. • Be able to use the result $Var\,(a + bX) = b^2\,Var\,(X)$ and understand its meaning. • Be able to find the expectation of any linear combination of independent random variables and the variance of any linear combination of independent random variables. • Recognise situations under which the discrete uniform distribution is likely to be an appropriate model. • Be able to calculate probabilities using a discrete uniform distribution. • Be able to calculate the mean and variance of any given discrete uniform distribution. 3. **Discrete Probability Distributions**: • Recognise situations under which the binomial distribution is likely to be an	resourcing and scheduling. • Be able to use a network to model a transmission • system. • Be able to specify a cut and calculate its capacity. • Understand and use the maximum flow/minimum cut theorem. • Understand that network algorithms can be explored, understood and tested in cases in which the algorithm can be run by hand, but for practical problems the algorithm needs to be formulated in a way suitable for computing power to be applied. 4. **Linear Programming**: • Understand and use the language associated with linear programming. • Be able to identify and define variables from a given problem. • Be able to formulate a problem as a linear program.	given value of the other and know when it is appropriate to do so. 2. **Bivariate, Chi Squared Tests**: • Be able to interpret bivariate categorical data in a contingency table. • Be able to apply the X^2 test (chi-squared) to a contingency table. • Be able to interpret the results of a X^2 test using tables of critical values or the output from software. • Be able to carry out a X^2test for goodness of fit of a uniform, binomial, or Poisson model. • Be able to interpret the results of a X^2 test using tables of critical values or the output from software. 3. **Matrices**: • Be able to find the determinant and inverse of a 3 × 3 matrix without a calculator. • Know the meaning of, and be able to find, invariant	resolving forces in suitable directions, or by drawing and using a polygon of forces. **3. A Model for Friction** • Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram. • Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction). • Understand that the frictional force may be modelled by $F > \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F = \mu R$ when sliding occurs. • Be able to derive and use the result that a body on a rough slope

	matrix gives the area scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image. • Know that the magnitude of the determinant of a 3 × 3 matrix gives the volume scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image. • Know that det(MN) = det M × det N and the corresponding result for scale factors of transformations. • Understand what is meant by an inverse matrix. • Be able to calculate the inverse of a non-singular 2 × 2 matrix or 3 × 3 matrix. • Be able to use the inverse of a non-singular 2 × 2 or 3 × 3 matrix. Relate the inverse matrix to the	• Be able to interpret and apply algorithms presented in • a variety of formats. • Be able to repair, develop and adapt given algorithms • Understand and be able to use the basic ideas of algorithmic complexity and be able to analyse the • complexity of given algorithms. • Know that complexity can be used, among other things, to compare algorithms. • Understand that algorithms can sometimes be proved correct or incorrect. • Understand and know the importance of heuristics. • Know and be able to use the quick sort algorithm. Be able to apply other sorting algorithms which are specified. • Be able to count the number of comparisons and/or swaps needed in particular applications of sorting algorithms, and relate this to complexity. • Be able to reason about a given sorting algorithm.	appropriate model, and be able to calculate probabilities to use the model. • Know and be able to use the mean and variance of a binomial distribution, $n = np$ and $v^2 = np\,(1 - p)$. Prove these results in particular cases. • Recognise situations under which the Poisson distribution is likely to be an appropriate model. • Recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models. • Be able to calculate probabilities using a Poisson distribution. • Know and be able to use the mean and variance of a Poisson distribution. • Know that the sum of two or more independent Poisson distributions is also	• Be able to recognise when an LP is in standard form • Be able to graph inequalities in 2-D and identify feasible regions. • Be able to recognise infeasibility. • Be able to solve a 2-D LP graphically. • Be able to consider the effect of modifying constraints or the objective function. • Be able to solve 2-D integer LP problems graphically. • Be able to use a visualisation of a 3-D LP to solve it. • Be able to reduce a 3-D LP to a 2-D LP when one constraint is an equality. 5. **Simplex Method**: • Be able to use the simplex algorithm on an LP in • augmented form. • Understand the geometric basis for the simplex • method. • Recognise that if an LP includes $ constraints then the two-stage	points and invariant lines for a linear transformation. 4. **Vectors**: • Be able to form and use the equation of a line in 2-D and 3-D. • Be able to calculate the angle between • two lines. • Be able to determine whether two lines in three dimensions are parallel, skew or intersect, and to find the point of intersection if there is one • Know the different ways in which two lines can intersect or not in 3-D space. • Be able to find the intersection of a line and a plane. • Be able to calculate the angle between a line and a plane.	inclined at an angle a to the horizontal is on the point of slipping if $\mu = tan\, a$. • Be able to apply Newton's laws to situations involving friction. **4. Dimensional Analysis** • Be able to find the dimensions of a quantity in terms of M, L, T. • Understand that some quantities are dimensionless. • Be able to determine the units of a quantity by reference to its dimensions. • Be able to change the units in which a quantity is given. • Be able to use dimensional analysis to check the consistency of a relationship. • Use dimensional analysis to determine unknown indices in a proposed formula. • Use a model based on dimensional analysis.

	corresponding inverse transformation. • Understand and use the product rule for inverse matrices.	• Know and be able to use first fit and first fit decreasing • packing algorithms and full bin strategies. • Be able to count the number of comparisons needed in particular applications of packing algorithms, and relate this to complexity.	a Poisson distribution. • Recognise situations under which the geometric distribution is likely to be an appropriate model. • Be able to calculate the probabilities within a geometric distribution, including cumulative probabilities. • Know and be able to use the mean and variance of a geometric distribution. 4. **Bivariate, Measuring Correlation**: • Understand what bivariate data are and know the conventions for choice of axis for variables in a scatter diagram. • Be able to use and interpret a scatter diagram. • Interpret a scatter diagram produced by software. • Be able to calculate the pmcc from raw data or summary statistics.	simplex method may be used; understand how this method works and be able to set up the initial tableau in such cases. • Be able to reformulate an equality constraint as a pair of inequality constraints. • Recognise that if an LP has variables which may take negative values or requires the objective function to be minimised then some initial reformulation is required before the simplex algorithm may be applied. • Be able to use slack variables to convert an LP in • standard form to augmented form. • Understand that some LPs can be solved using graphical techniques or the simplex method, but for practical problems computing power needs to be applied.		

			• Know when it is appropriate to carry out a hypothesis test using Pearson's product moment correlation coefficient. • Be able to carry out hypothesis tests using the *pmcc* and tables of critical values or the *p-value* from software. • Use the *pmcc* as an effect size • Be able to calculate Spearman's rank correlation coefficient from raw data or summary statistics. • Be able to carry out hypothesis tests using Spearman's rank correlation coefficient and tables of critical values or the output from software. • Decide whether a test based on r or rs may be more appropriate, or whether neither is appropriate.	• Know that a spreadsheet LP solver routine, or other software, can solve an LP given in standard form or, in some cases, in non-standard form. • Be able to interpret the output from a spreadsheet optimisation routine, or other software, for the simplex method or ILPs.		
Understanding / Sequence of delivery	1. Building on prior knowledge and making connections between topics.					

Assessment	End of Topic Assessed Homework Exam Style Questions Grade Boundaries based on A Level 2019	End of Topic Assessed Homework and Practice Papers Exam Style Questions Grade Boundaries based on A Level 2019	Practice Papers Grade Boundaries based on A Level 2019
	POP Test Past Exam Questions Grade Boundaries based on A Level 2019	POP Test Past Exam Questions Grade Boundaries based on A Level 2019	PPE Past Exam Questions Grade Boundaries based on A Level 2019